Regression-Based Control Charts for Detecting Anomalies in Crop Yields

Muhammad Noor-ul-Amin

Department of Statistics, COMSATS University Islamabad, Lahore, Pakistan.
(nooramin.stats@gmail.com)

Abstract

This study proposes a novel application of regression-based control charts (RCC) to monitor agricultural yield performance in Pakistan. While control charts are widely utilized in manufacturing and healthcare, their use in agriculture remains limited. By integrating statistical process control techniques with agronomic data, we aim to identify abnormal variations in crop yields that deviate from expected behavior after accounting for key influencing factors. Using 25 years (1999–2023) of data on wheat and rice yields, rainfall, and fertilizer consumption—along with optional variables such as temperature and irrigation—we develop multiple linear regression models to estimate yield based on climatic and input variables. Residuals from these models are then plotted on regression control charts to flag years where actual yield significantly diverged from predicted levels. These "out-of-control" signals are further investigated through qualitative analysis of historical events, including pest outbreaks, policy changes, and natural disasters. This mixed-method approach offers a robust framework for agricultural monitoring and can support timely decision-making for food security and resource management in South Asia.

Keywords: Statistical Quality Control; Crop Yield; Average run length

1. Introduction

Statistical Process Control (SPC) offers an organization of the process stability monitoring and identification of the exits of the in-control state. Shewhart, Cumulative Sum (CUSUM) and Exponentially Weighted Moving Average (EWMA) are classical control charts that are common in identifying assignable causes on process performance. Some assumptions that are normally made in the development of these charts include the independence and equal distributions of process observations with constant mean and variance. But in most practice cases, this is not the case, since the quality characteristic of interest is an effect of a one or more explanatory variables. By disregarding this auxiliary information, one might receive misleading signals, overreported false alarms, or miss actual process changes (Montgomery, 2019). To overcome this drawback, control charts which are based on regression have been designed, where the systematic influences of the known covariates are eliminated by initial regression modelling and a subsequent monitoring of the regression residuals is then done. The ground breaking study of Haworth et al (2006) proved that the regression-adjusted control charts greatly enhance monitoring performance in situations whereby the process mean is systematically dependent on explanatory variables. CUSUM charts based on regression have since been used successfully in manufacturing, chemical processing, environmental surveillance and in agricultural systems, where covariate effects are important in forming process behaviour (Noorossana et al., 2011). Regression-based CUSUM charts are especially useful in identifying the gradual and long-term changes in the underlying process mean as the unexplained variability is isolated. Although they have benefits, the vast majority of currently available regression based CUSUM charts are based on predetermined charting parameters, which presupposes that the size of the possible process shift is priori. Practically, however, this is hardly ever the case. In complex systems such as agricultural production, environmental processes, and policy-driven systems, shift magnitudes may be unknown, time-varying, or heterogeneous. These variations may arise due to climatic variability, resource limitations, pest outbreaks, or regulatory interventions, time varying, or heterogeneous magnitudes due to such factors as climatic variability, resource limitations, pest outbreaks or regulatory interventions. In this state of affairs, a fixed reference CUSUM chart can react too slowly to significant deviations or be too sensitive to natural variations to give an early warning or a false alarm. Such challenges have driven the creation of adaptive control charting schemes, where the important chart parameters are dynamically adjusted, according to an observed process behavior. Adaptive CUSUM charts have been demonstrated to be more efficient in detecting changes of large magnitudes of shifts and offer a compromise between sensitivity and stability in a data adaptive way (Reynolds et al., 1988; Capizzi and Masarotto, 2003). Nonetheless, although adaptive mechanisms have been studied extensively in the standard SPC context, there is little literature on the integration of adaptive mechanisms with regression-adjusted CUSUM monitoring especially in systems that have covariate-based processes.

Having been inspired by this gap, the current study suggests a RA-ACUSUM control chart to monitor the process mean with the presence of the explanatory variables. The suggested method uses an in-control regression model to estimate the systematic effects of the covariates of interest and subsequently applies an adaptive CUSUM scheme to the standardized regression residuals. The RA-ACUSUM chart is dynamic, unlike the classical regression CUSUM charts, which use fixed reference values because the reference parameter changes dynamically based on the magnitude of observed residual deviations. Such a dynamic is achieved by this adaptive process which causes the chart to act conservatively when the in-control conditions are almost near but it becomes more sensitive as evidence of a permanent shift gathers. The efficiency of the suggested RA-ACUSUM chart is verified by using the extensive Monte Carlo simulation experiments and compared with benchmark regression-based CUSUM and regression-based EWMA charts. Competing schemes are adjusted to have the same in-control Average Run Length (ARL) so that they can be fair and meaningful compared with each other. The results of the simulation indicate that the suggested RA-ACUSUM chart shows better results in detecting small- to medium-scale shifts and preserving the in-control behavior, which makes it especially suitable in the contexts where the early recognition of the slightest types of deviations is essential.

The rest of the paper is structured in the following way. Section 2 examines the regression-based CUSUM model. The proposed RA-ACUSUM chart and the adaptive mechanism are introduced in Section 3. Section 4 gives the simulation study and analysis of comparative performance. Section 5 demonstrates how the suggested chart can be applied in reality based on data about agricultural production, and Section 6 is the conclusion part of the paper with some final words and orientation on the further research.

2. Regression-Based Control Charts

Classical statistical process control techniques are primarily designed for monitoring processes in which observations are assumed to be independent and identically distributed with constant mean and variance. In many practical applications, however, the process output is systematically influenced by one or more explanatory variables. When such covariate effects are ignored, conventional control charts may generate misleading signals, increase false alarm rates, or fail to detect meaningful process shifts in a timely manner. To address these challenges, regression-based control charts have been developed, in which the relationship between the quality characteristic and relevant explanatory variables is explicitly modeled prior to monitoring.

2.1. Regression Adjustment in Statistical Process Control

The central idea of regression-based control charts is to separate systematic variation attributable to known covariates from unexplained process variation. Let Y_t denote the process output observed at time t , and let $X_t = \left(x_{1t}, x_{2t}, \dots, x_{tp}\right)^T$ represent a vector of explanatory variables. Under in-control conditions, the process can be modeled as

$$Y_t = x_t^T \beta + \epsilon_t \tag{1}$$

where x_t denotes the vector of explanatory variables. β is the corresponding vector of regression coefficients, and ϵ_t represents a random error term assumed to have mean zero and constant variance. The regression parameters are estimated using Phase-I (in-control) data. The regression model is fitted to give the anticipated process output.

$$\hat{Y}_t = x_t^T \hat{\beta} \tag{2}$$

The corresponding standardized regression residual is defined as

$$Z_t = \frac{Y_t - \hat{Y}_t}{\hat{\sigma}} \tag{3}$$

where $\hat{\sigma}$ denotes the estimated standard deviation of the residuals. Under nominal operating conditions, the standardized residuals $\{Z_t\}$ are assumed to follow a standard normal distribution.

After regression adjustment is done, the residual sequence is monitored instead of monitoring the original observations

3. Proposed Regression-Based Adaptive CUSUM Chart

Based on the classical regression CUSUM model described in Section 2, the present study introduces RA-ACUSUM control chart that is aimed at becoming more sensitive to changes of different sizes. The suggested chart adds a dynamic process to the CUSUM framework whereby the reference value varies dynamically over time in answer to any observed deviation in the regression residuals. The RA-ACUM chart enhances the performance of regression adjustment with adaptive accumulation wherein both small and moderate shifts are better detected with desirable in-control characteristics. The following are the methodological details of the proposed chart.
Let $\mathrm{Y_t}$ denote the observed crop yield or production at time t. Agricultural output is influenced by several systematic factors such as cultivated area, fertilizer usage, and input intensity. To account for these effects, a regression model is first fitted using historical in-control data

$$Y_t = \beta_0 + \beta_1 X_{\{1t\}} + \beta_2 X_{\{2t\}} + \cdots + \beta_p X_{\{pt\}} + \epsilon_t \tag{4}$$

where $X_{\{jt\}}$represents the j^{th} explanatory variable (e.g., area harvested, nitrogen, phosphorus, potash), and $\epsilon_t$is a random error term with mean zero and variance σ^2. The fitted model provides the expected output

$$\hat{Y}_t = \hat{\beta}_0 + \sum_{j=1}^{p} \beta_j X_{jt} \tag{5}$$

And the corresponding standardized regression residual is defined as

$$z_t = \frac{Y_t - \hat{Y}_t}{\hat{\sigma}_\varepsilon} \tag{6}$$

where$\hat{\sigma}_\varepsilon$ is the estimated standard deviation of the residuals. Under in-control conditions, z_t is assumed to follow a standard normal distribution. Monitoring is then performed on the residual sequence z_t , rather than on the raw observations, thereby isolating anomalies that cannot be explained by known covariates. The classical CUSUM chart uses a predefined reference value k which implicitly assumes that the amount of the process shift is known upfront. But within an agricultural system the exogenous variation of the planned production could occur due to a wide range of unpredictable causes which can include weather extremes, pest outbreaks, or policy actions, or even discontinuities in the supply-chain. As a result, a fixed reference can result in either a high rate of false alarms or delayed response. We assume that an adaptive reference value that varies dynamically with the size of the standardized regression residual can be used to overcome this limitation. This enables the chart to differentiate routine variance and substantive variation in an empirical way. Adaptive reference value at time t is given as.

$$k_t = k_{min} + (k_{max} - k_{min})\big(1 - exp(-\alpha|z_t|)\big) \tag{7}$$

where: k_{min}> 0 represents the minimum reference value corresponding to sensitivity for small shifts, k_{max} >k_{min}represents tolerance to routine noise, and $\alpha > 0$ controls the rate of adaptation. This formulation ensures that k_t increases monotonically with the magnitude of $|z_t|$allowing conservative behavior under minor fluctuations and aggressive response under pronounced deviations. Using the adaptive reference value, the one-sided CUSUM statistics are defined as:

$$C_t^+ = max(0, C_{t-1}^+ + z_t - k_t) \tag{8}$$

$$C_t^- = max(0, C_{t-1}^- - z_t - k_t) \tag{9}$$

with initial values$C_0^+ = C_0^- = 0$. The two-sided RA-ACUSUM chart signals an out-of-control condition whenever

$$max(C_t^+, C_t^-) > h$$

where h is the decision interval. Because the reference value is adaptive, there are no analytical expressions of the in-control average run length ARL_0. Thus, Monte Carlo simulation is used to tune the decision interval h so that it results in a nominal ARL_0 of a given value (e.g., 370). Fig 1 illustrates the operational steps of the proposed RA-ACUSUM monitoring scheme.

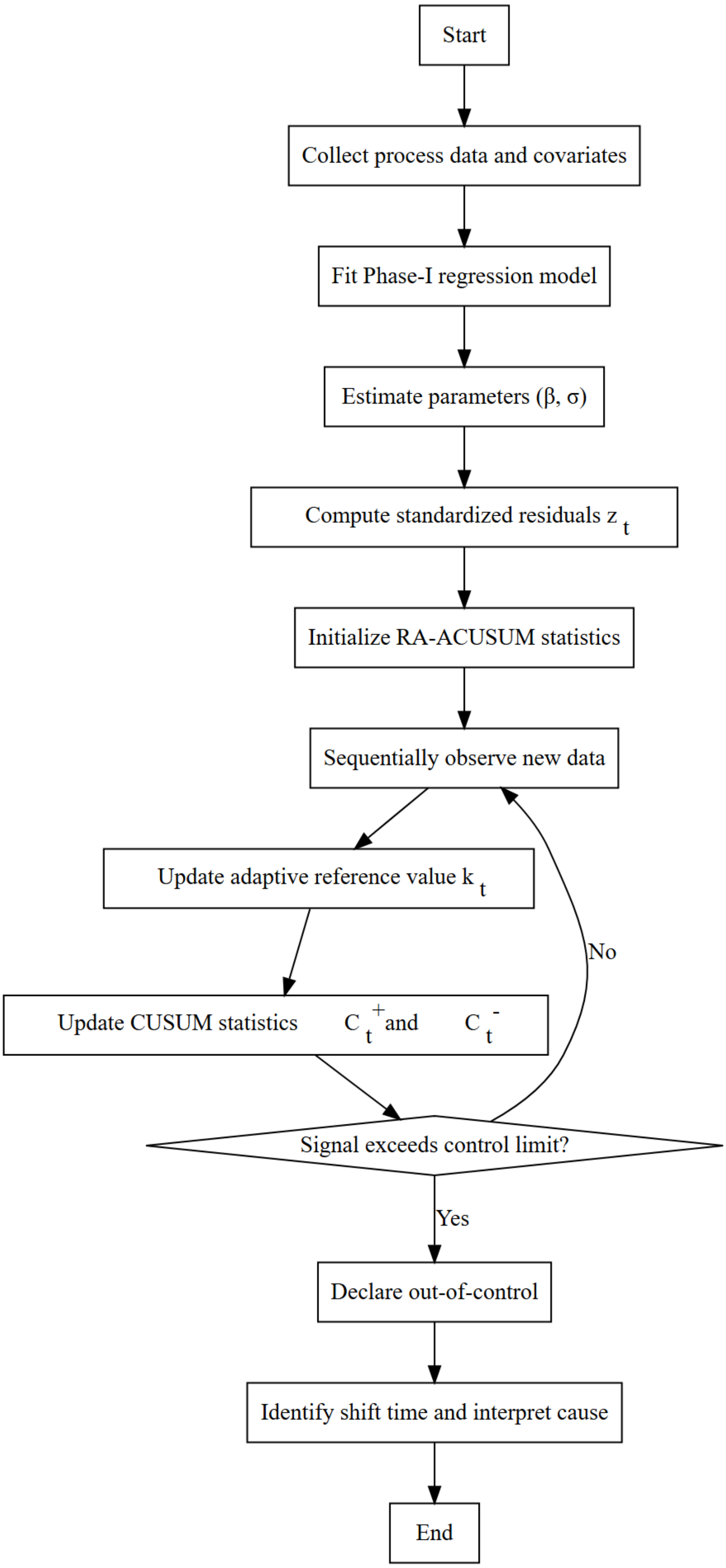

Fig 1. Flowchart illustrating the implementation steps of the proposed RA-ACUSUM monitoring scheme.

4. Simulation Study

This part is the Monte Carlo simulation study that will be applied to test the effectiveness of the suggested RA-ACUSUM chart. All the results reported in Tables 1-3 are generated by utilizing the following simulation steps.

1. Generate Phase-I observations with $\delta = 0$ and estimate parameters of the regression model.
2. Get standardized residuals z_t of the Phase-I fitted model.
3. Create Phase-II of observations sequentially and add mean displacement of size δ.
4. Using the Phase-II residuals, update one-sided statistics, $C_t^+$and C_t^-at each time point.
5. Proclaim out of control signal when the monitoring statistic is more than the decision interval and update the run length RL.
6. Repeat a large amount of replications and calculate ARL, SDRL, and run-length percentiles of choice.

4.1 Performance Evaluation

The effectiveness of the proposed RA-ACUSUM chart is examined with a massive Monte Carlo simulation study. The test is based on the in-control stability and out-of-control detection efficiency of a broad magnitude of mean shift. The run length (RL) distribution is characterized by the Average Run Length (ARL), the Standard Deviation of Run Length (SDRL), and the selected percentiles (P05, P25, P50, P75, and P95) and all of them are used to give a comprehensive description of the speed and variation of detection.

Every scheme is tuned to nominal in-control ARL of about 370 so that a fair and meaningful comparison between various rates of adaptation is obtained. Tables 1-3 show the run length profiles of the proposed RA-ACUSUM chart under three of the typical values of the adaptation parameter i.e., a = 0.15, 0.30 and 0.45. The proposed chart has great in-control behavior at all values of a. Table 1-3 illustrates that the in-control ARLs are always close to the desired level of 370 and the SDRLs and percentile spreads are similar. This shows that the adaptive reference mechanism would not overinflate the false alarm rate and that the calibration of the decision interval has been successfully done in simulation. In the case of moderate changes, the chart shows a sharp increase in the speed of detection with an average RL under 25 and median values under 20. There is also a significant decrease in the widths of the interquartile ranges indicating more stable signal behavior and less ambiguity in the time of detection. In the case of large shifts, the proposed chart gives almost instantaneous detection, and ARLs reach their theoretical minimum. Here, the SDRL as well as the percentile spread collapses quickly, and it means that there is very strong predictable performance once the process is significantly out of control. The main aspect of the suggested RA-ACUSUM chart is the flexibility that is proposed by the parameter of adaptation a. Comparison of Tables 1-3 indicates that increasing results in moderately and large shift being detected relatively faster at the expense of some slight shift in aggressiveness. Nevertheless, the disparities between them are small, and all environments save the nominal in-control ARL having similar percentile designs. It means that the offered adaptive mechanism is not too sensitive to the selection and can be adjusted to strike the balance between responsiveness and robustness depending on the preferences of the practitioners. The percentile based analysis, in addition to the average performance, shows the positive distributional characteristics of the suggested chart. The steady decrease in upper percentile (P75 and P95) with d is evidence that the RA-ACUSUM does not only decrease the expected detection delay, but the extreme delays are also constrained. It is a good practical asset, because it guarantees an opportune signaling even with counter favorable random realizations.

Table 1: Run length profile of the proposed control chart for various mean shift magnitudes (δ) and adaptation rates (α=0.15)

δ	ARL	SDRL	P05	P25	P50	P75	P95
0	372.46	361.54	31	116	262	514	1102
0.1	253.79	238.37	25	85	181	347	737
0.2	125.4	112.16	18	46	91	169	354
0.4	41.9	30.26	11	21	34	54	102
0.6	21.84	12.52	8	13	19	27	46
0.8	14.49	6.72	6	10	13	18	27
1	10.78	4.29	5	8	10	13	19
1.5	6.65	2.06	4	5	6	8	10
2	4.85	1.26	3	4	5	6	7
2.5	3.85	0.89	3	3	4	4	5
3	3.22	0.68	2	3	3	4	4
3.5	2.79	0.58	2	2	3	3	4
4	2.44	0.52	2	2	2	3	3
5	2.05	0.26	2	2	2	2	3

Table 2: Run length profile of the proposed control chart for various mean shift magnitudes (δ) and adaptation rates (α=0.30)

δ	ARL	SDRL	P05	P25	P50	P75	P95
0	371.65	365.93	29	113	257	510	1099
0.1	260.79	251.34	24	82	183	361	760
0.2	133.43	123.53	16	46	95	181	380
0.4	43.7	33.58	10	20	34	57	111
0.6	22.27	13.71	8	13	19	28	49
0.8	14.33	7.25	6	9	13	18	28
1	10.48	4.51	5	7	10	13	19
1.5	6.27	2.04	4	5	6	7	10
2	4.57	1.26	3	4	4	5	7
2.5	3.63	0.88	2	3	4	4	5
3	3.02	0.69	2	3	3	3	4
3.5	2.59	0.58	2	2	3	3	3
4	2.28	0.47	2	2	2	3	3
5	1.98	0.26	2	2	2	2	2

Table 3: Run length profile of the proposed control chart for various mean shift magnitudes (δ) and adaptation rates (α=0.45)

δ	ARL	SDRL	P05	P25	P50	P75	P95
0	371.86	361.12	28	114.75	263	509	1082
0.1	264.67	255.85	22	83	189	363	773
0.2	141.61	132.14	16	47.75	102	194	399
0.4	46.54	38	10	20	36	61	121
0.6	22.43	14.63	7	12	19	28	51
0.8	14.27	7.55	6	9	13	18	29
1	10.24	4.64	5	7	9	13	19
1.5	6.08	2.1	3	5	6	7	10
2	4.38	1.26	3	3	4	5	7
2.5	3.43	0.88	2	3	3	4	5
3	2.86	0.68	2	2	3	3	4
3.5	2.44	0.56	2	2	2	3	3
4	2.17	0.42	2	2	2	2	3
5	1.89	0.34	1	2	2	2	2

5. Comparative Analysis

A comparative run length profile of the proposed RA-ACUSUM chart is compared to the Regression-Based CUSUM (RBCUSUM) chart and Regression-Based EWMA (RBEWMA) chart in table 4. Comparison will be made under the same in-control conditions and all charts will be calibrated in order to have in-control average run length of about 370 and so that the performance can be registered fairly and meaningfully. With the in-control state ($\delta = 0$), the three charts have almost the same value of the ARL and similar interquartile range values, which confirms that the adaptive structure of the RA-ACUSUM chart does not exaggerate the false alarm rate. This would prove the statistical stability of the proposed chart and prove its calibration process. The proposed RA-ACUSUM chart always works best compared to the competing charts in the case of small and middle-range shifts in the mean. Specifically, it has significantly smaller ARLs and smaller median (P50) and upper-quartile (P75) run lengths. This means that it not only detects more quickly on average, but also more reliably and predictably. The gains are also large in comparison with the RBEWMA benchmark, which demonstrates the benefits of the introduction of adaptive smoothing into the regression-adjusted EWMA model. As compared to the RBCUSUM chart, the RA-ACUSUM is more sensitive to minor changes which is particularly critical in agricultural surveillance where early changes in crop yield are more likely to occur in a gradual than a sudden manner. The larger the magnitude of the shift ($\delta = 0.8$), the closer the performance gap of the charts gets. In moderate-to-large shifts, RBCUSUM chart is a bit more efficient in the ARL and median run length as it is known to be optimally suited to large sustained shifts. In the meantime, the RBEWMA chart is the quickest to detect very large shifts ($\delta = 2$), but has worse robustness and worse aggressiveness, as indicated by its interquartile range collapsing very rapidly. These findings are supported by theoretical characteristics of the memory-type control charts and also by the complementary nature EWMA-based and CUSUM-based designs. Altogether, the findings in Table 4 show that the suggested RA-ACUSUM chart can be characterized as a balanced and strong monitoring solution, especially in the sense of detecting small-to-moderate anomalies without losing any in-control behavior. The proposed chart can be particularly applicable to crop yield surveillance and policy-driven agricultural monitoring systems where the timely observation of the initial signs of anomalies is essential to timely act and make decisions.

Table 4: Comparative run length profiles of the proposed (RA-ACUSUM), (RBCUSUM), and (RBEWMA) control charts under various mean shift magnitudes, based on ARL and interquartile percentiles (P25, P50, P75)

δ	Proposed Adaptive Chart ARL	P25	P50	P75	CUSUM Chart ARL	P25	P50	P75	EWMA Chart ARL	P25	P50	P75
0.0	372.46	116	262	514	372.70	110	263	509	370.97	104	253	502
0.1	253.79	85	181	347	286.09	83	198	395	288.63	81	200	398
0.2	125.40	46	91	169	164.13	51	114	225	177.46	51	122	245
0.4	41.90	21	34	54	53.94	20	40	73	61.76	20	44	84
0.6	21.84	13	19	27	24.63	12	19	32	27.37	10	20	37
0.8	14.49	10	13	18	14.40	8	12	18	15.28	7	12	20
1.0	10.78	8	10	13	9.87	6	9	12	9.47	5	8	12
1.5	6.65	5	6	8	5.52	4	5	7	4.49	3	4	6
2.0	4.85	4	5	6	3.86	3	4	5	2.79	2	3	4
2.5	3.85	3	4	4	3.01	2	3	3	1.99	1	2	2
3.0	3.22	3	3	4	2.49	2	2	3	1.56	1	1	2
3.5	2.79	2	3	3	2.17	2	2	2	1.30	1	1	2
4.0	2.44	2	2	3	1.95	2	2	2	1.14	1	1	1
5.0	2.05	2	2	2	1.62	1	2	2	1.02	1	1	1

6. Case Study

An example of the real agricultural data is used to demonstrate the practical use of the proposed RA-ACUSUM control chart. The annual data on rice production in Pakistan were accessed through the Food and Agriculture Organization (FAO) FAOSTAT database to obtain internationally harmonized and publicly available agricultural statistics as the source, as far as longitudinal agricultural analysis is concerned, is considered to be credible.

The rice production (Y_t) was hypothesized as a product of the vital agronomic inputs, i.e the area harvested under rice (X_{1t}), the consumption of nitrogen (X_{2t}), phosphorus (X_{3t}) and potassium (X_{4t}). Variables were synchronized to a similar time frame and considered a series of processes and therefore were appropriate to statistical process monitoring. Speaking of the conventional SPC practice, the data was separated into Phase-I (baseline) and Phase-II (monitoring) intervals. Phase-I was assigned approximately 65% of the observations and the rest of the data were set aside to be used in Phase-II monitoring. During Phase-I, multiple linear regression model was estimated to describe the production of rice in terms of agronomic inputs:

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \beta_3 X_{3t} + \beta_4 X_{4t} + \varepsilon_t \quad (10)$$

where $\varepsilon_t \sim N(0, \sigma^2)$

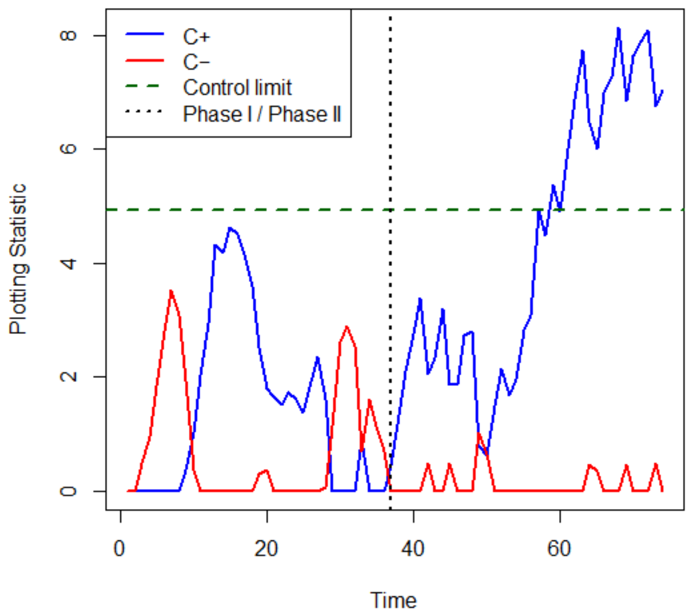

Fig 2: RA-ACUSUM chart for rice production monitoring. Phase-I data are used to estimate the regression model and establish the in-control baseline, while shifted data (δ =0.25) are monitored sequentially for departures from the baseline.

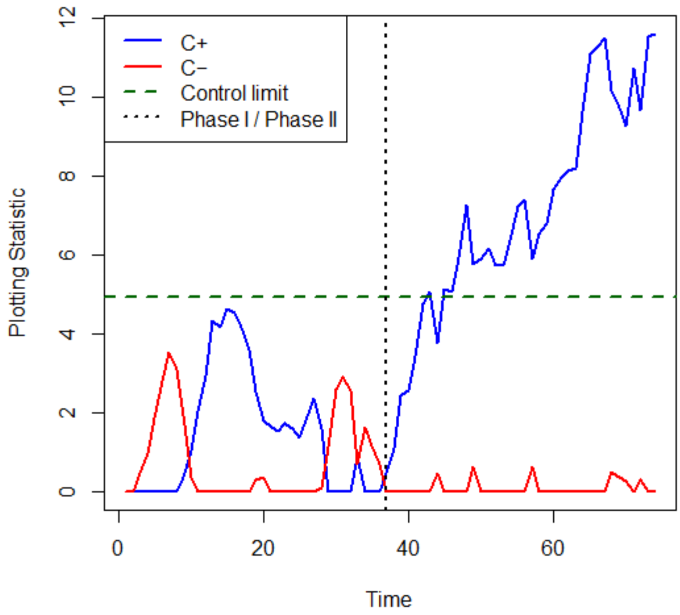

Fig 3: RA-ACUSUM chart for rice production monitoring. Phase-I data are used to estimate the regression model and establish the in-control baseline, while shifted data (δ =0.5) are monitored sequentially for departures from the baseline.

The model fitted represents the anticipated behavior of the production of rice when systematic effects of the cultivated area and the use of fertilizers are taken into account. The standardized residuals of the Phase-I regression model as will be adopted in the proposed RA-ACUSUM methodology, were then taken as the input to construct control charts. All parameters of charting were chosen according to the rules explained above, and the goal of the charting was to depict how the proposed approach performs as opposed to maximizing its performance on this particular data set. During Phase-II, the regression model estimated in Phase-I was held and new observations were observed sequentially by use of RA-ACUSUM chart. The example shows how the offered RA-ACUSUM chart can be applied into reality with the help of real data and how it will help to identify the events of leaving the defined regression structure. The example is intended solely for methodological illustration and does not aim to draw substantive conclusions regarding rice production dynamics.

Figs 2 and 3 depict how the proposed RA-ACUSUM control chart can be applied to the data of rice production in Pakistan where the model of rice production is considered to depend on the use of agronomic factors such as cultivated area and the consumption of fertilizers. The fitted regression model using Phase-I data gives the in-control baseline, and standardized residual is followed sequentially in Phase-II to detect the non-conformity of the desired production behavior even after controlling influencing covariates. Fig 2 shows the performance of the RA-ACUSUM chart in a moderate scenario of the mean shift ($\delta = 0.25$). It is observed that the chart is within control limits in the first stage of Phase-I, which proves that the fitted regression model is adequate and that the process in the base is stable. The RA-ACUSUM statistic indicates a slow build-up of evidence on the introduction of the shift in Phase-II, and an out-of-control signal is triggered in a relatively short time. This act proves how the proposed chart can identify small to moderate deviations that cannot be attributed to the explanatory variables known to us. Such deviations can be reflective of unobservable conditions in the agricultural systems such as pest outbreaks, policy interferences, market shocks or unquantified climatic anomalies. Fig 3 shows the results of the monitoring with a larger shift magnitude ($\delta = 0.5$). In this example, the RA-ACUSUM value increases at a faster rate and indicates nearly immediately the beginning of the shift, which indicates the high sensitivity of the chart to sharp variations. The detection delay is significantly smaller than in the moderate shift case, and this is in line with the results of the simulation discussed in the earlier part of the paper.

In general, Figs 2 and 3 reveal the usefulness of the RA-ACUSUM chart in the monitoring of agriculture. The chart removes confounding effects of covariates using regression analysis, which means that unusual changes in crop production are identified early and abnormal conditions are given early warning. The case study findings support the simulation findings that the suggested chart provides a fair performance in terms of increased sensitivity to slight and moderate shifts and rapid identification of big shifts, which makes it a promising device in real-time agricultural monitoring and policymaking decision support.

7. Conclusion

This paper suggested a RA-ACUSUM control chart to be used as a monitoring method of processes affected by measurement variables when the size of the possible shifts is unknown. The proposed method balances sensitivity and stability by using regression adjustment with an adaptive CUSUM structure. Simulation experiments had shown that RA-ACUSUM chart exhibited stable in-control behaviour and was able to detect small to moderate shifts at a faster rate compared to traditional regression based CUSUM and EWMA charts. The fact that it was applied to agricultural production data also demonstrated its practical usefulness. In general, the suggested chart can be used as a versatile and efficient monitoring tool of covariate-driven processes with the possibility to further extend to more sophisticated regression frameworks and dependent data, thus representing possibilities of the future studies.

References

1. Montgomery DC. *Introduction to statistical quality control*. 8th ed. Hoboken (NJ): John Wiley & Sons; 2019.
2. Haworth DA, Cheng SW, Wang K. Regression-adjusted control charts. J Qual Technol. 2006;38(1):78–89.
3. Noorossana R, Saghaei A, Amiri A. *Statistical analysis of profile monitoring*. Hoboken (NJ): John Wiley & Sons; 2011.
4. Reynolds MR Jr, Amin RW, Arnold JC, Nachlas JA. CUSUM charts with variable sampling intervals. Technometrics. 1988;30(2):161–73.
5. Capizzi G, Masarotto G. An adaptive exponentially weighted moving average control chart. Technometrics. 2003;45(3):199–207.
6. AlQadi H, Abdelfattah W, Abbas T, et al. Design of an EWMA control chart by adaptation of smoothing constant based on a function of estimated shift. Sci Rep. 2025;15:24730.
7. Arshad A, Noor-ul-Amin M, Hanif M. Function-based adaptive exponentially weighted moving average dispersion control chart. Qual Reliab Eng Int. 2021;37(6):2325–48.
8. Noor-ul-Amin M, Arshad A, Hanif M. A function-based adaptive EWMA mean monitoring control chart. Qual Reliab Eng Int. 2022;38(1):1–65.
9. Neto JCL, Gomes MI, Figueiredo F. Process monitoring using robust regression control charts. Qual Reliab Eng Int. 2025;41(1):1–15.
10. Kazmi MW, Noor-ul-Amin M. Integrating machine learning–based EWMA control charts for multivariate process monitoring. Comput Ind Eng. 2025;204:111131.