Asymptotic Relative Efficiency of Randomized Designs

Abimibola Victoria Oladugba
Department of Statistics, University of Nigeria, Nsukka

Corresponding Author: abimibola.oladugba@unn.edu.ng

Abstract
Relative efficiency is a measure used to assess the effectiveness of blocking in experimental designs. This paper develops, and evaluates the asymptotic relative efficiency of randomized complete block designs compared to completely randomized designs, based on multiple comparison procedures. The asymptotic relative efficiency is expressed as the ratio of the expected squared half-lengths of multiple comparison procedures (Tukey, Bonferroni, and Scheffé simultaneous confidence limits). A simulation study was used to demonstrate the applicability of the developed methods. he results showed that randomized complete block designs were more efficient than completely randomized designs across almost all the different values of treatments and blocks considered in this study.
Keywords: Relative efficiency, multiple comparison, confidence interval, completely randomized design, randomized complete block design

1. Introduction

The completely randomized (CR) and randomized complete block (RCB) designs are among the simplest and most useful statistical experimental designs for making inferences about the differences in treatment means. The CR design is primarily used when the experimental units are more homogeneous, and extraneous factors are easily randomized under controlled environments. Conversely, when extraneous factors are known but cannot be controlled, RCB designs are preferred. RCB designs minimize variability within blocks and maximize it across blocks. The advantage of RCB design over CR design, in terms of increasing precision, lies in reducing experimental error. The magnitude of this reduction can be determined by computing the relative efficiency (RE) of the two designs. This reduction in experimental error leads to a decrease in experimental error degrees of freedom (df) for hypothesis testing and multiple comparisons, resulting in less sensitive comparisons depending on the RE of RCB design to CR design (Oladugba et al., 2013; Shieh & Jan, 2004; Vonesh, 1983).

Relative efficiency of RCB design to CR design is the number of replications necessary to reach the same level of precision in terms of the experimental error of a difference between treatment means (Oladugba et al., 2013; Shieh & Jan, 2004; Abou-El-Fittouh, 1976). Many studies have investigated the RE of RCB design compared to CR design, including but not limited to Oladugba et al. (2022), Bhat et al. (2019), Khan (2017), Lohmor et al. (2017), Omer et al. (2014), Mudi and Usman (2012), Shieh and Jan (2004), and Abou-El-Fittouh (1976). Recent research continues to explore various aspects of the relative efficiency of RCB designs. For instance, Sanadya et al. (2023) examined the effectiveness of alpha lattice designs over RCB designs in agricultural trials, demonstrating the superior efficiency of lattice designs under certain conditions. Moreover, Shami and Mhemdi (2023) introduced neutrosophic analysis to address missing values in augmented RCB designs, reflecting ongoing innovation in experimental design methodologies.

These studies are based on the experimental errors of the two designs, where the design with smaller experimental error usually results in a gain in efficiency. Several other modified measures of relative efficiency of randomized designs have been suggested in the literature (see Okeke, et al., 2022; Oladugba et al., 2013; Shieh & Jan, 2004; Samuel et al., 1991 & 1994; Vonesh, 1983; Jensen, 1980 & 1982; Morrison, 1972). These modified measures, referred to as asymptotic relative efficiency (ARE), do not account for the loss in degrees of freedom due to blocking. However, there has been no reported work on the ARE of randomized complete block design to completely randomized design in the available literature. This study aims to fill this gap by introducing and evaluating ARE measures for randomized designs.

This paper develops, and evaluates three asymptotic relative efficiency measures for assessing the efficiency of randomized complete block designs relative to completely randomized designs. These ARE measures, based on multiple comparison procedures (Tukey, Bonferroni, and Scheffe), are derived in terms of the expected squared half-length of confidence intervals from these procedures. A simulation study was conducted to demonstrate the applicability of the developed measures.

The remainder of this paper is organized as follows. Section 2 presents the statistical model of CR and RCB designs. Section 3 discusses the three procedures of asymptotic relative efficiency. Section 4 provides an illustrative example and discussion, and Section 5 concludes the paper.

2. Statistical Model of CR and RCB Designs

Suppose an experiment was conducted using CR design in testing the effect of t - treatments replicated b - times on tb – homogeneous experimental units on a single response variable, Y. Let Y_{ij} represent the observed response from the j^{th} replication of the i^{th} treatments, $i = 1, 2, \ldots, t$ and $j = 1, 2, \ldots, b$; μ, τ_i, and ε_{ij} represent the overall mean, the effect of i^{th} treatments and experimental error associated with the observed response, Y_{ij}. Then, the statistical linear model for CR design under the normality assumption is given by (1).

$$Y_{ij} = \mu + \tau_i + \varepsilon_{ij} \quad ; \varepsilon_{ij} \sim N(0, \sigma^2_{CR}) \,. \tag{1}$$

Similarly, suppose the experiment was conducted in a RCB design with t-treatments replicated r-times within b-blocks. Let Y_{ij} represent the observed response from the j^{th} block of the i^{th} treatments, $i = 1, 2, \ldots, t$ and $j = 1, 2, \ldots, b$; and μ, τ_i, β_j and ε_{ij}, represent the overall mean, the effect of i^{th} treatments, the effect of the j^{th} block and experimental error associated with the observed response, Y_{ij}.

The statistical linear model for RCB design is defined as

$$Y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij} \;, \tag{2}$$

where the ε_{ij}'s are independently, distributed $N(0, \sigma^2_{RCB})$.

The null hypothesis ($H_0 : \mu_1 = \ldots = \mu_t$) of no significant difference in treatment effect under the CR design and RCB design models given in (1) and (2), can be tested using the F-statistics defined by (3) and (4), respectively.

$$F_{CR} = t(b-1)\sum_i (\bar{Y}_{i.} - \bar{Y}_{..})^2 \Big/ (t-1)\sum_{ij} (\bar{Y}_{ij} - \bar{Y}_{i.})^2 \ , \tag{3}$$

$$F_{RCB} = (t-1)(b-1)\sum_i (\bar{Y}_{i.} - \bar{Y}_{..})^2 \Big/ (t-1)\sum_{ij} (\bar{Y}_{ij} - \bar{Y}_{i.} - \bar{Y}_{.j} + \bar{Y}_{..})^2 \ , \tag{4}$$

where, F_{CR} and F_{RCB} follow F-distribution with (t-1) and t(b-1), and (t-1) and (t-1)(b-1) degrees of freedom; Y_{ij} is the individual observations; $\bar{Y}_{i.}$ is the mean number of observations per treatment; $\bar{Y}_{.j}$ is the mean number of observations per block and $\bar{Y}_{..}$ is the overall mean.

3. Asymptotic Relative Efficiency of RCB Designs to CR Designs

In this section, the asymptotic relative efficiency (*ARE*) of RCB designs to CR designs in Tukey, Bonferroni and Scheffe multiple comparison procedures were derived based on the models given in Section 2. The *ARE* is obtained in terms of the ratio of the expected squared half-length of Tukey, Bonferroni and Scheffé simultaneous confidence intervals. The RCB designs are more efficient than the CR designs if and only if for any pairwise $d = (\mu_i - \mu_{i'})$, or g contrast comparisons at a fixed α, the ratio of the expected squared half lengths is less than or equals to one.

3.1 Tukey Procedure

The Tukey procedure is used when all pairwise comparisons predetermined before observing the data are of interest. It provides narrower confidence intervals than that of the Bonferroni procedure.

Let $d = (\mu_i - \mu_{i'})$ for all $i \neq i'$, be the set of all pairwise comparisons of treatment means. The $(1 - \alpha)$ 100% Tukey simultaneous confidence interval on treatment mean difference $(\mu_i - \mu_{i'})$ for all $i \neq i'$, for CR and RCB designs has expected squared half-length, denoted by $E_{CR}(T, \alpha)$ and $E_{RCB}(T, \alpha)$, given by (5) and (6), respectively.

$$E_{CR}(T,\alpha) = (4b)^{-1}\sigma_{CR}^2 (\sum_{i=1}^{t} |c_i|)^2 q_\alpha^2 (t, t(b-1)) \tag{5}$$

$$E_{RCB}(T,\alpha) = (4b)^{-1}\sigma_{RCB}^2 (\sum_{i=1}^{t} |c_i|)^2 q_\alpha^2 (t, (t-1)(b-1)) \tag{6}$$

where $q_\alpha^2(t, t(b-1))$ and $q_\alpha^2(t, (t-1)(b-1))$ are from the Studentized range distribution with parameters t and t(b-1), and t and (t-1)(b-1) for CR and RCB designs respectively.

The *ARE* in terms of Tukey procedure denoted by *ARET*, is obtained from the ratio of the expected squared half-lengths defined in (5) and (6). This is shown in (7).

$$ARET = \frac{E_{RCB}(T,\alpha)}{E_{CR}(T,\alpha)} = \frac{\sigma_{RCB}^2 q_\alpha^2 (t, (t-1)(b-1))}{\sigma_{CR}^2 q_\alpha^2 (t, t(b-1))} \leq 1 \tag{7}$$

3.2 Bonferroni Procedure

The Bonferroni procedure is used for evaluating a few of the *g* possible pairwise comparison of interest selected prior (predetermined) to observing the data. This procedure is better than the Scheffe procedure when the number of contrasts of interest is less than or equals to the number of treatments.

Let $g = \frac{t(t-1)}{2}$ denote the number of predetermined pairwise comparisons to be made on the treatment means. The (1 – α)100% Bonferroni simultaneous confidence interval on treatment mean difference $(\mu_i - \mu_{i'})$ for all $i \neq i'$, for CR and RCB designs has expected squared half-length, denoted by $E_{CR}(B, \alpha)$ and $E_{RCB}(B, \alpha)$, given by (8) and (9), respectively.

$$E_{CR}(B,\alpha) = (b)^{-1} 2\sigma_{CR}^2 \sum_{i=1}^{t} c_i^2 t_{(1-\alpha/2g, t(b-1))}^2 \tag{8}$$

$$E_{RCB}(B,\alpha) = (b)^{-1} 2\sigma_{RCB}^2 \sum_{i=1}^{t} c_i^2 t_{(1-\alpha/2g, (t-1)(b-1))}^2 \tag{9}$$

where $t_{(1-\alpha/2g, t(b-1))}^2$ and $t_{(1-\alpha/2g, (t-1)(b-1))}^2$ are from Student's *t* probability distribution with parameters $t(b$-1) and $(t$-1)$(b$-1).

The *ARE* in terms of the expected squared half-length of Bonferroni simultaneous confidence interval denoted by *AREB*, is defined in (10).

$$AREB = \frac{E_{RCB}(B,\alpha)}{E_{CR}(B,\alpha)} = \frac{\sigma_{RCB}^2 t_{(1-\alpha/2g, (t-1)(b-1))}^2}{\sigma_{CR}^2 t_{(1-\alpha/2g, t(b-1))}^2} \leq 1 \tag{10}$$

3.3. Scheffé Procedure

Scheffé procedure is applied in estimating all possible *g* contrasts among the treatment means after the data have been observed. It is preferred when many or all contrasts are of interest.

Let $g = \sum_{i=1}^{t} c_i \mu_i$, where $\sum_{i=1}^{m} c_i = 0$, be the sets of family of interest of all possible contrasts among treatment effects. The (1 – α) 100% Scheffé simultaneous confidence interval on $g = \sum_{i=1}^{t} c_i \mu_i$ contrasts for CR and RCB designs has expected squared half-length, denoted by $E_{CR}(S, \alpha)$ and $E_{RCB}(S, \alpha)$, given by (11) and (12), as

$$E_{CR}(S,\alpha) = (b)^{-1} 2\sigma_{CR}^2 \sum_{i=1}^{t} c_i^2 (t-1) F_\alpha((t-1), t(b-1)) \tag{11}$$

$$E_{RCB}(S,\alpha) = (b)^{-1} 2\sigma_{RCB}^2 \sum_{i=1}^{t} c_i^2 (t-1) F_\alpha((t-1), (t-1)(b-1)) \tag{12}$$

where $F_\alpha((t-1), t(b-1))$ and $F_\alpha((t-1), (t-1)(b-1))$ are from the F-distribution with parameters $(t$ – 1) and $t(b$ – 1), and $(t$ – 1) and $(t$ -1)$(b$-1) respectively.

The *ARE* in terms of Scheffé procedure denoted by *ARES*, formed in the ratio of the expected squared half-lengths is defined in (13).

$$ARES = \frac{E_{RCB}(S,\alpha)}{E_{CR}(S,\alpha)} = \frac{\sigma^2_{RCB} F_\alpha((t-1),(t-1)(b-1))}{\sigma^2_{CR} F_\alpha((t-1),t(b-1))} \leq 1 \quad (13)$$

Remark: The RCB designs will be more efficient than the CR designs for a given α, and any pairwise or contrast comparisons if the largest value of *ARE* in (7), (10) and (13) does not exceed one.

4. Simulation Study

Let $\lambda = \frac{\sigma^2_{RCB}}{\sigma^2_{CR}}$ in equations (7), (10) and (13). The *ARE* of RCB designs to CR designs in terms of *ARET*, *AREB* and *ARES* for several values of t, b, g and λ, are presented in Tables 1, 2, and 3.

It was observed that the RCB designs were more efficient than the CR designs in almost all the values of λ except in some cases in Tables 2 and 3 particularly at $\lambda = 0.90$ where the values exceed one. This implies that small values of λ lead to considerable gains in efficiency for RCB designs over the CR designs. Although the asymptotic measure indicates greater efficiency of the RCB designs; an RCB design should not be used without a clear definition of which interferent is being blocked. This is important because it is a common mistake to recommend RCB in experimental field designs, even in flat environments, without a clear separation of blocks perpendicular to the direction of the interfering variable.

Table 1: ARE of RCB designs to CR designs based on 95% Tukey confidence intervals

		λ				
Number of blocks (b)	Number of treatments (t)	0.10	0.25	0.50	0.75	0.90
	3	0.1071	0.2678	0.5355	0.8033	0.9639
	4	0.1038	0.2595	0.5189	0.7784	0.9340
5	5	0.1024	0.2560	0.5120	0.7679	0.9215
	6	0.1017	0.2541	0.5083	0.7624	0.9149
	3	0.1029	0.2574	0.5147	0.7721	0.9265
	4	0.1016	0.2540	0.5080	0.7620	0.9144
10	5	0.1011	0.2526	0.5053	0.7579	0.9095
	6	0.1007	0.2519	0.5037	0.7556	0.9067
	3	0.1018	0.2546	0.5092	0.7637	0.9165
	4	0.1010	0.2525	0.5051	0.7576	0.9091
15	5	0.1007	0.2517	0.5033	0.7550	0.9059
	6	0.1005	0.2512	0.5023	0.7535	0.9041
	3	0.1014	0.2534	0.5068	0.7601	0.9122
	4	0.1008	0.2519	0.5038	0.7556	0.9068
20	5	0.1005	0.2512	0.5024	0.7536	0.9043
	6	0.1004	0.2509	0.5019	0.7528	0.9033
	3	0.1011	0.2527	0.5053	0.7580	0.9095
	4	0.1006	0.2514	0.5029	0.7543	0.9051
25	5	0.1004	0.2510	0.5019	0.7529	0.9034
	6	0.1003	0.2507	0.5014	0.7520	0.9024

Table 2: ARE of RCB designs to CR designs based on 95% Bonferroni confidence intervals

Number of Blocks(b)	λ	Number of treatments (t)											
		$t = 3$			$t = 4$			$t = 5$			$t = 6$		
		$g = 2$	$g = 5$	$g = 10$	$g = 2$	$g = 5$	$g = 10$	$g = 2$	$g = 5$	$g = 10$	$g = 2$	$g = 5$	$g = 10$
	0.10	0.1155	0.1061	0.1028	0.1072	0.1030	0.1014	0.1042	0.1018	0.1008	0.1027	0.1012	0.1005
	0.25	0.2888	0.2653	0.2569	0.2679	0.2574	0.2534	0.2604	0.2544	0.2520	0.2568	0.2529	0.2514
5	0.50	0.5776	0.5306	0.5138	0.5359	0.5148	0.5068	0.5208	0.5088	0.5041	0.5136	0.5058	0.5027
	0.75	0.8664	0.7959	0.7707	0.8038	0.7722	0.7602	0.7811	0.7631	0.7561	0.7703	0.7586	0.7541
	0.90	1.0397	0.9551	0.9248	0.9645	0.9266	0.9122	0.9374	0.9158	0.9073	0.9244	0.9104	0.9049
	0.10	0.1079	0.1035	0.1250	0.1038	0.1017	0.1112	0.1022	0.1010	0.1064	0.1015	0.1007	0.1041
	0.25	0.2698	0.2588	0.3124	0.2595	0.2543	0.2779	0.2556	0.2526	0.2659	0.2537	0.2517	0.2603
10	0.50	0.5397	0.5177	0.6248	0.5190	0.5087	0.5558	0.5112	0.5052	0.5318	0.5074	0.5034	0.5206
	0.75	0.8095	0.7765	0.9372	0.7785	0.7630	0.8336	0.7668	0.7577	0.7976	0.7610	0.7551	0.7808
	0.90	0.9714	0.9318	1.1246	0.9342	0.9156	1.0004	0.9202	0.9093	0.9572	0.9132	0.9061	0.9370
	0.10	0.1038	0.1049	0.1057	0.1019	0.1024	0.1028	0.1011	0.1014	0.1016	0.1007	0.1009	0.1011
	0.25	0.2595	0.2622	0.2644	0.2547	0.2560	0.2570	0.2528	0.2535	0.2541	0.2518	0.2523	0.2527
15	0.50	0.5190	0.5245	0.5287	0.5093	0.5119	0.5139	0.5055	0.5071	0.5082	0.5037	0.5047	0.5054
	0.75	0.7785	0.7867	0.7931	0.7640	0.7679	0.7709	0.7583	0.7606	0.7624	0.7555	0.7570	0.7582
	0.90	0.9343	0.9440	0.9517	0.9168	0.9214	0.9251	0.9100	0.9127	0.9148	0.9066	0.9084	0.9098
	0.10	0.1041	0.1296	0.1109	0.1020	0.1130	0.1052	0.1012	0.1074	0.1030	0.1008	0.1048	0.1020
	0.25	0.2604	0.3240	0.2772	0.2551	0.2826	0.2629	0.2530	0.2684	0.2575	0.2520	0.2619	0.2550
20	0.50	0.5207	0.6481	0.5544	0.5101	0.5651	0.5258	0.5060	0.5369	0.5151	0.5040	0.5238	0.5099
	0.75	0.7811	0.9721	0.8316	0.7652	0.8477	0.7886	0.7590	0.8053	0.7726	0.7560	0.7856	0.7649
	0.90	0.9373	1.1665	0.9979	0.9182	1.0172	0.9464	0.9108	0.9663	0.9271	0.9072	0.9428	0.9178
	0.10	0.1022	0.1028	0.1032	0.1011	0.1014	0.1016	0.1006	0.1008	0.1009	0.1004	0.1005	0.1006
	0.25	0.2554	0.2569	0.2581	0.2527	0.2534	0.2540	0.2516	0.2520	0.2524	0.2511	0.2513	0.2516
25	0.50	0.5108	0.5138	0.5162	0.5053	0.5068	0.5079	0.5032	0.5041	0.5047	0.5021	0.5027	0.5031
	0.75	0.7662	0.7708	0.7743	0.7580	0.7602	0.7619	0.7548	0.7561	0.7571	0.7532	0.7540	0.7547
	0.90	0.9195	0.9249	0.9291	0.9096	0.9123	0.9143	0.9057	0.9073	0.9085	0.9038	0.9048	0.9056

Table 3: ARE of RCB designs to CR designs based on 95% Scheffé confidence intervals

Number of blocks (b)	Number of treatments (t)	λ				
		0.10	0.25	0.50	0.75	0.90
	3	0.1148	0.2869	0.5739	0.8608	1.0329
	4	0.1078	0.2694	0.5388	0.8082	0.9698
5	5	0.1049	0.2623	0.5246	0.7868	0.9442
	6	0.1034	0.2586	0.5172	0.7758	0.9310
	3	0.1060	0.2650	0.5299	0.7949	0.9538
	4	0.1033	0.2582	0.5164	0.7746	0.9295
10	5	0.1021	0.2553	0.5106	0.7659	0.9191
	6	0.1015	0.2538	0.5076	0.7613	0.9136
	3	0.1037	0.2594	0.5187	0.7781	0.9337
	4	0.1021	0.2552	0.5104	0.7656	0.9187
15	5	0.1014	0.2534	0.5068	0.7602	0.9122
	6	0.1010	0.2524	0.5049	0.7573	0.9087
	3	0.1027	0.2568	0.5136	0.7704	0.9245
	4	0.1015	0.2538	0.5076	0.7614	0.9137
20	5	0.1010	0.2525	0.5050	0.7575	0.9090
	6	0.1007	0.2518	0.5036	0.7553	0.9064
	3	0.1021	0.2554	0.5107	0.7661	0.9193
	4	0.1012	0.2530	0.5060	0.7590	0.9108
25	5	0.1008	0.2520	0.5040	0.7559	0.9071
	6	0.1006	0.2514	0.5028	0.7542	0.9050

5. Conclusion

In this paper, the asymptotic relative efficiency measure of RCB designs to CR designs based on multiple comparison procedures was proposed and evaluated. The measure was derived in terms of the Tukey, Bonferroni and Scheffé simultaneous confidence intervals expected squared half-length. The results from the simulation study showed that the RCB designs were more efficient than the CR designs in almost all the different treatment and block values. Therefore, we recommend that the asymptotic relative efficiency measures obtained in this paper can be used in evaluating the efficiency of RCB designs relative to the CR designs in situation where the loss in df due to blocking is not accounted for or otherwise.

References

Abou-el-Fittouh, H. (1976). Relative efficiency of the randomized complete block square design. *Experimental Agriculture, 12*(2), 145-149.

Bhat, I. A., Mir, S.A., Nazir, N., Khan, I., Wani, J. A. & Shah, I. (2019). Does aspect-based blocking affect the experimental efficiency. *Multilogic in Science*, *8*(28). 122-123.

Jensen, D. R. (1980), Efficiencies in multivariate paired experiments, *Biometrical Journal 22*, 399-405.

Jensen, D. R. (1982), Efficiency and robustness in the use of repeated measurements, *Biometrics 38*, 813-825.

Morrison, D. F. (1972), The analysis of a single sample of repeated measurements. *Biometrics 28*, 55-71.

Khan, M., Hasija, R., Hooda, B. K., Tanwar, N. and Kumar, B. (2017). Relative efficiency of experimental designs in relation to various size and shape of plots and blocks in Indian Mustard (Brassica juncea L.) crop. *International Journal of Agricultural and Statistical Sciences*, *13*, 253-258.

Lohmor, N., Khan, M., Kapoor, K., Kumar and Tripathi, R. (2017). Studies on the relative efficiency of different experimental designs for sunflower (Helianthus annuus L.). *Annual Research & Review in Biology*, *12*(4): 1-7.

Mudi, A.T., & Usman, A. (2012). The Effect of blocking on the experimental precision of tomato yield. *Research Journal of Mathematics and Statistics*, *4*(1): 10-13.

Okeke, F. E., Oladugba, A. V., Onwuamaeze, U. C., & Asogwa, C. O. (2022). Space-filling orthogonal array based composite designs. *Journal of Econometrics and Statistics*. 2(2), 187-202.

Oladugba, A. V., Babatunde, T. O., Onuoha, D. O. and Opara, P. N. (2013). Relative efficiency of split plot design (SPD) to randomized complete block design (RCBD). *West African Journal of Industrial and Academic Research*, *7*(1): 113-120.

Oladugba, A. V., Babatunde, T. O. and Adeleke, M. L. (2022). Relative efficiency of Latin square design to randomized designs. *International Journal of Mathematics and Mathematical Science*, *23*(2): 31-40.

Omer, S. O., Osman, S. and Mahgoub, B. M. (2014). Comparing efficiency of on-farm experiments relative to designed experiments in complete blocks. *International Journal of Innovation and Applied Studies*, *8*(1): 2028-9324.

Shieh, G. and Jan, S. L. (2004). The effectiveness of randomized complete block design. *Statistica*

Neerlandica, *58*(1): 111-124.

Samuels, M. L., Casella, G. and McCabe, G. P. (1991). Interpreting blocks and random factors. *Journal of the American Statistical Association*, *86*, 798-821.

Samuels, M. L., Casella, G. and McCabe, G. P. (1994). Evaluating the efficiency of blocking without assuming compound symmetry. *Journal of Statistical Planning and Inference*, *38*, 237-248.

Sanadya, S. K., Sharma, R., & Jindal, M. (2023). Relative efficiency of alpha lattice and RCB designs. *International Journal of Agricultural Science, 15*(1), 85-93.

Shami, T. M., & Mhemdi, A. (2023). Neutrosophic analysis in augmented RCB designs. *Journal of Applied Statistical Science, 32*(2), 45-58.

Vonesh, E. F. (1983). Efficiency of repeated measures designs versus completely randomized designs based on multiple comparisons. *Communications in Statistics–Theory and Methods*, *12*, 289-301.